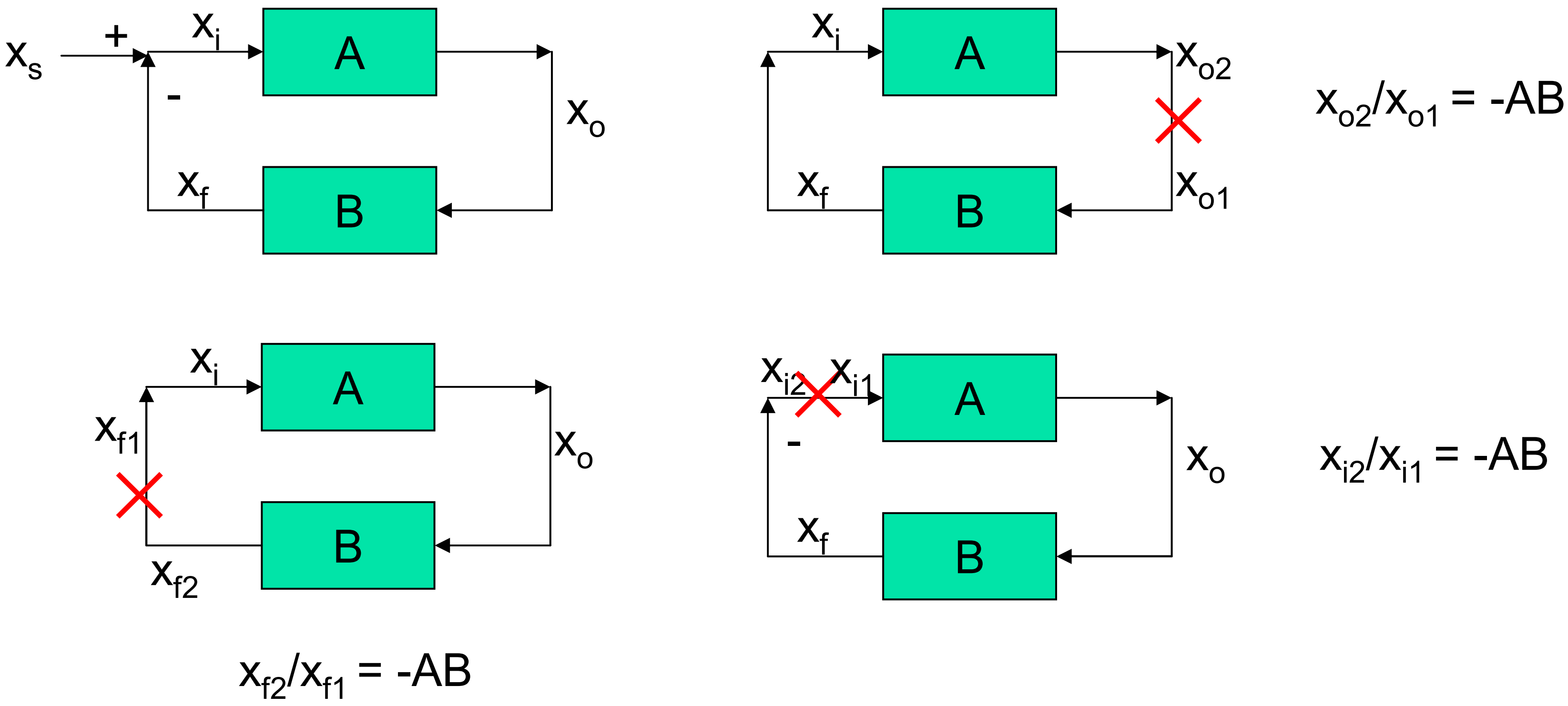
# 전자 회로 2

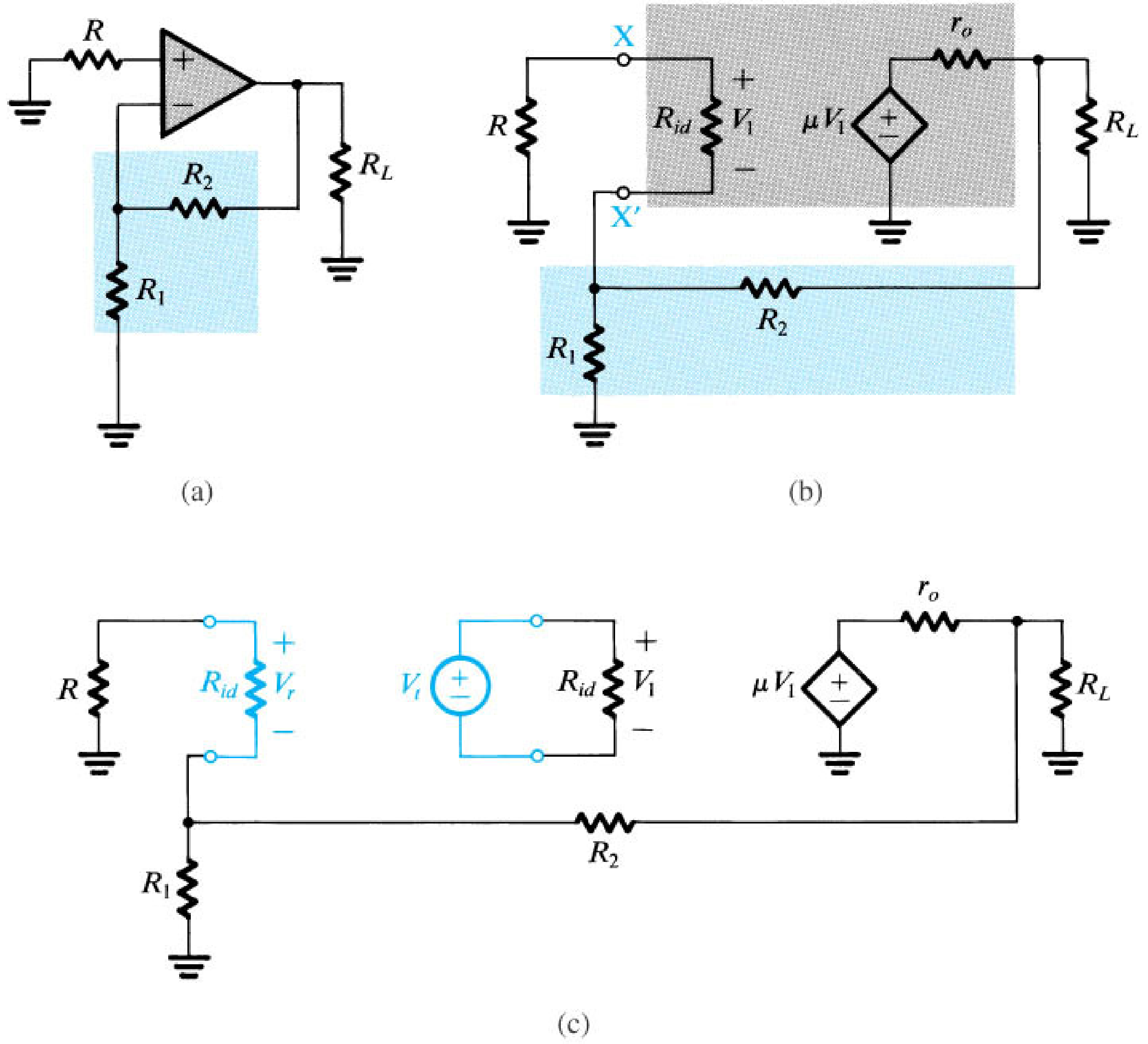
## Lecture 11 (Stability)

# Loop gain (AB)

- Loop gain: $x_s$=0일 때의 gain
- Loop 중 아무곳이나 끊어도 gain은 언제나 -AB가 된다.



# Loop gain (Example)



# Stability

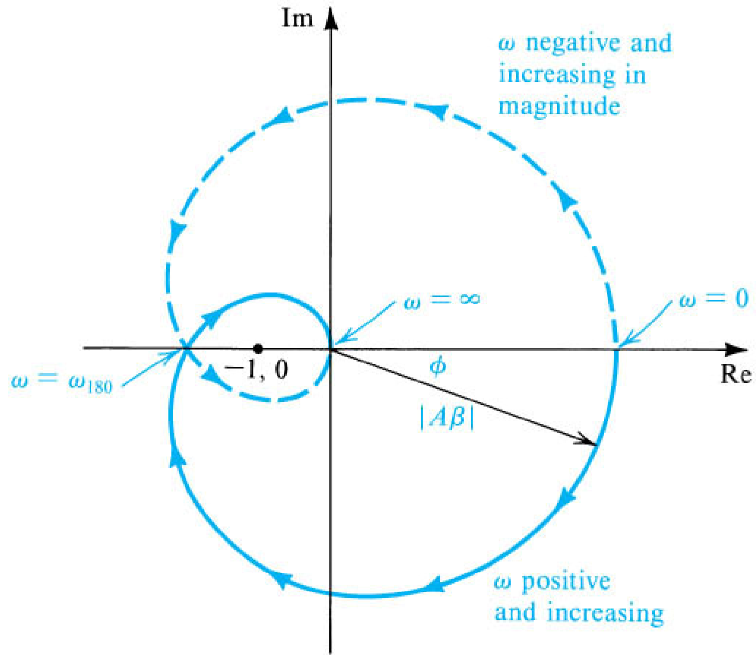
- $x_s = 0$일 때 loop gain L(jw)

$$A_f(s) = \frac{A(s)}{1 + A(s)B(s)}$$

$$\begin{aligned} L(s) &= A(s)B(s) \\ L(jw) &= A(jw)B(jw) \\ &= |A(jw)B(jw)|e^{j\phi(w)} \\ &= L(jw)e^{j\phi(w)} \end{aligned}$$

- $\phi(w) = 180^o$일 때 $L(jw) = -L_{180}$
- $L_{180} < 1$ ➔ stable (noise reduction)
- $L_{180} \geqq 1$ ➔ unstable (noise enhancement)

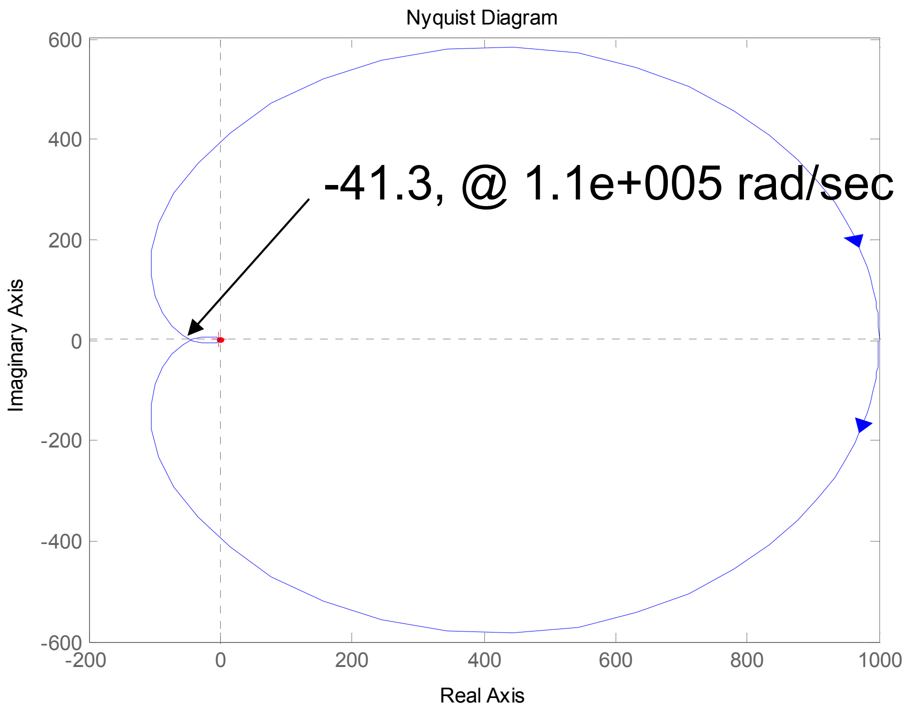
- Nyquist plot
  - 주파수에 따른 loop gain의 plot



# Ex.) Problem 8.64

$$A(s) = \frac{1000}{(1+s/10^4)(1+s/10^5)^2}$$

```
> A = zpk([], [-10^4, -10^5, -10^5], 10^17);
> nyquist(A)
```

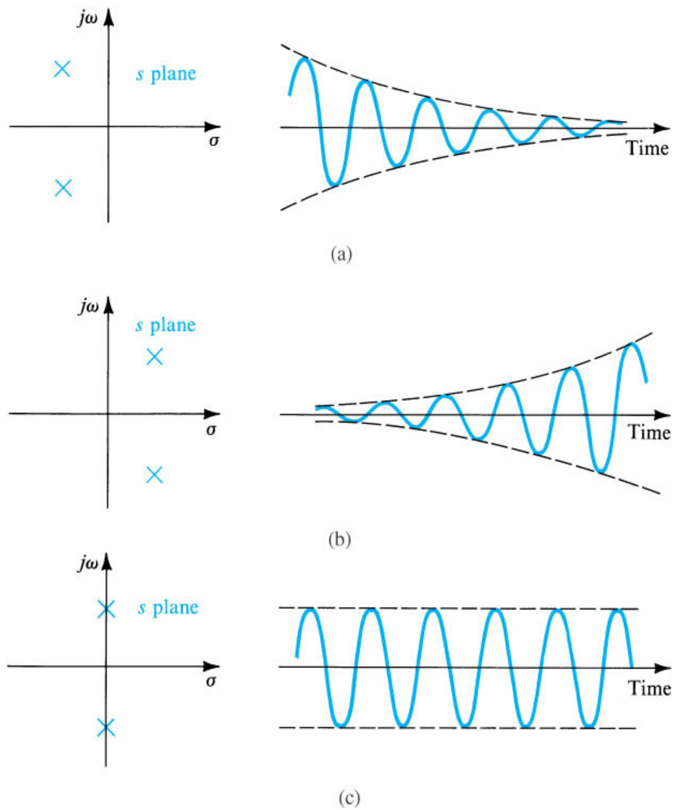



Oscillation이 없으려면 AB의 nyquist plot이 Re축과 만나는 점이 -1보다 작아야 한다.

➔ B < 1/41.3 ➔ system이 안정

# Stability & pole location

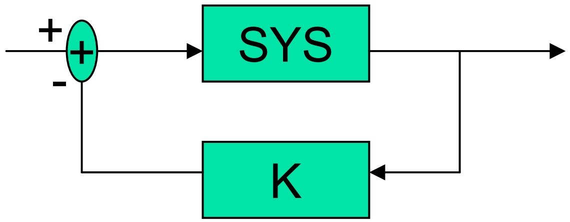
- In *s*-domain (Laplace transform)




- Poles of a feedback amp.
- Characteristic equation
  - 1+A(s)B(s) = 0
  - solution ➔ poles (s-domain)
- Pole이 left half plane에 있으면 시스템이 안정

# Root Locus (rlocus)

RLOCUS(SYS) computes and plots the root locus of the single-input, single-output LTI model SYS. The root locus plot is used to analyze the negative feedback loop




and shows the trajectories of the closed-loop poles when the feedback gain K varies from 0 to Inf. RLOCUS automatically generates a set of positive gain values that produce a smooth plot.

RLOCUS(SYS,K) uses a user-specified vector K of gain values.

# Case 1) A(s) has a single pole

$$A(s) = \frac{A_0}{1+s/w_p}$$

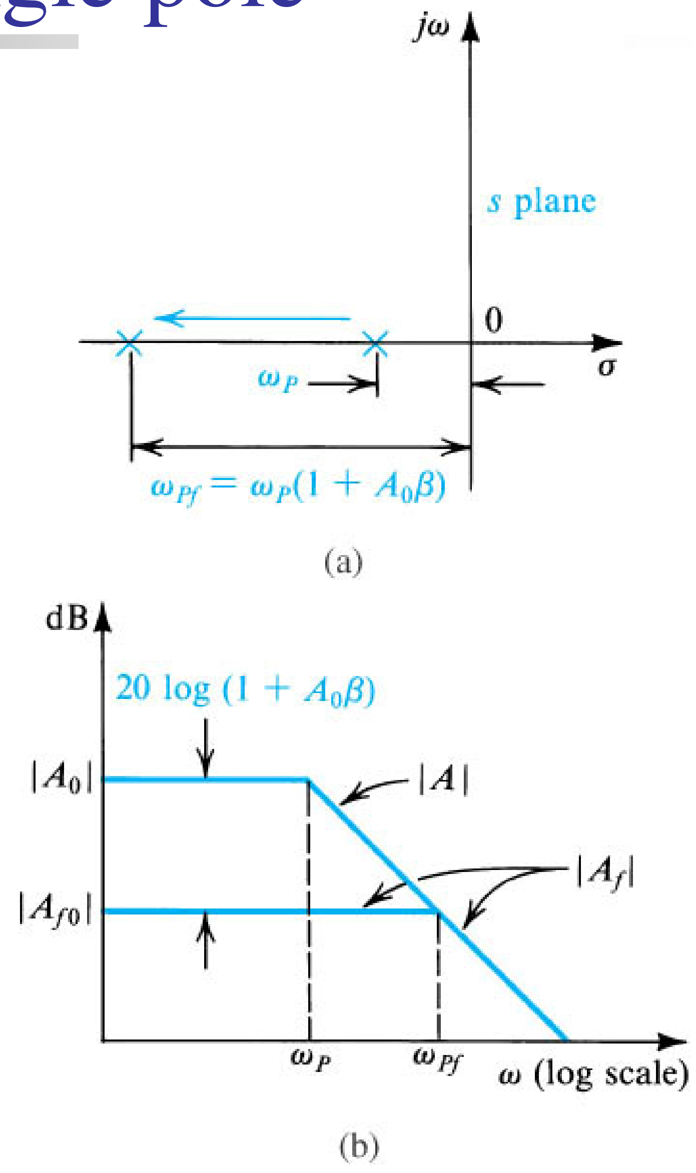
$$A_f(s) = \frac{A_0/(1+s/w_p)}{1+A_0B/(1+s/w_p)}$$

$$= \frac{A_0}{1+A_0B+s/w_p}$$

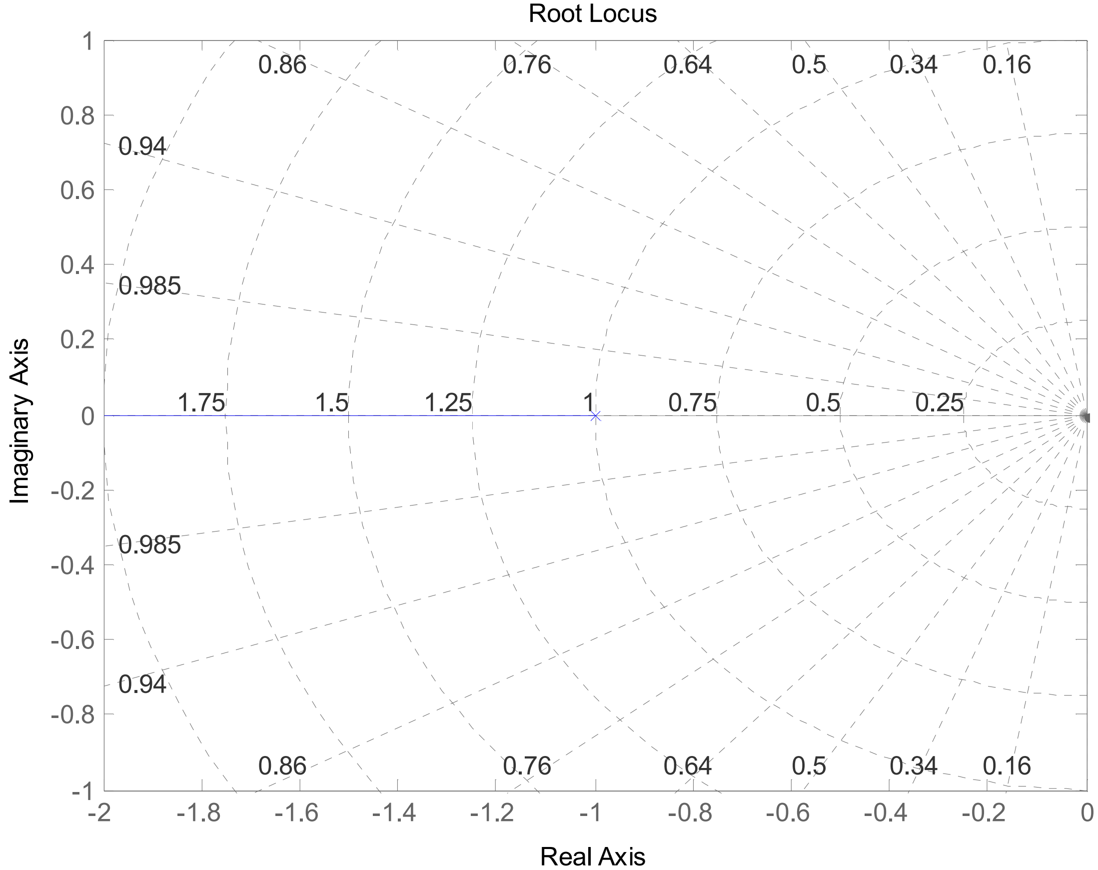
$$= \frac{A_0}{1+A_0B}\frac{1}{1+s/w_p(1+A_0B)}$$

$$w_{pf} = w_p(1+A_0B)$$

If basic amp. is stable
➔ feedback is stable



```
> A = tf(1, [1,1]);
> rlocus(A)
```

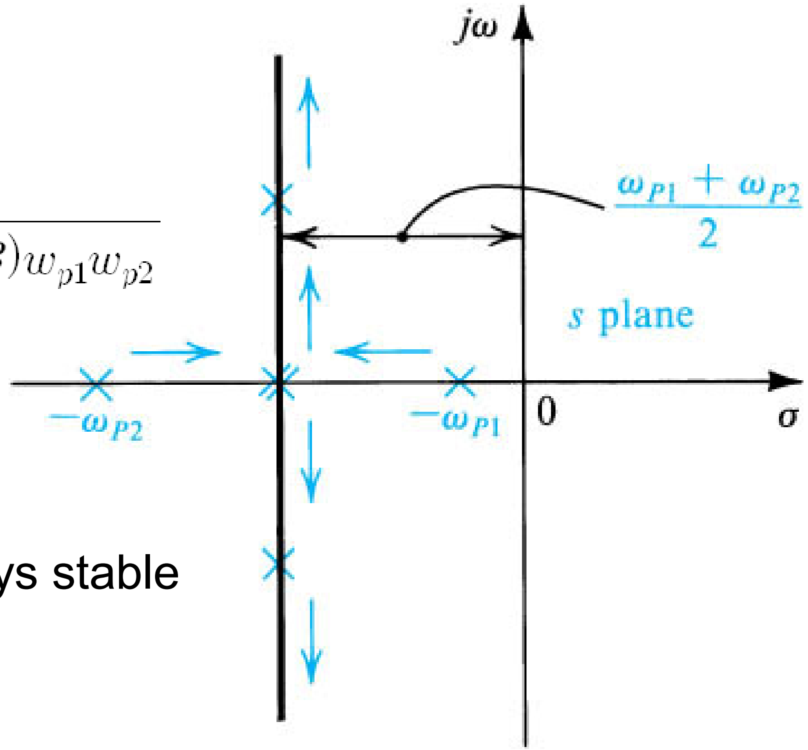


# Case 2) A(s) has two poles

$$A(s) = \frac{A_0}{(1 + s/w_{p1})(1 + s/w_{p2})}$$

$$s^2 + s(w_{p1} + w_{p2}) + (1 + A_0 B) w_{p1} w_{p2} = 0$$

$$s = -\frac{w_{p1} + w_{p2}}{2} \pm \frac{1}{2}\sqrt{(w_{p1} + w_{p2})^2 - 4(1 + A_0 B) w_{p1} w_{p2}}$$

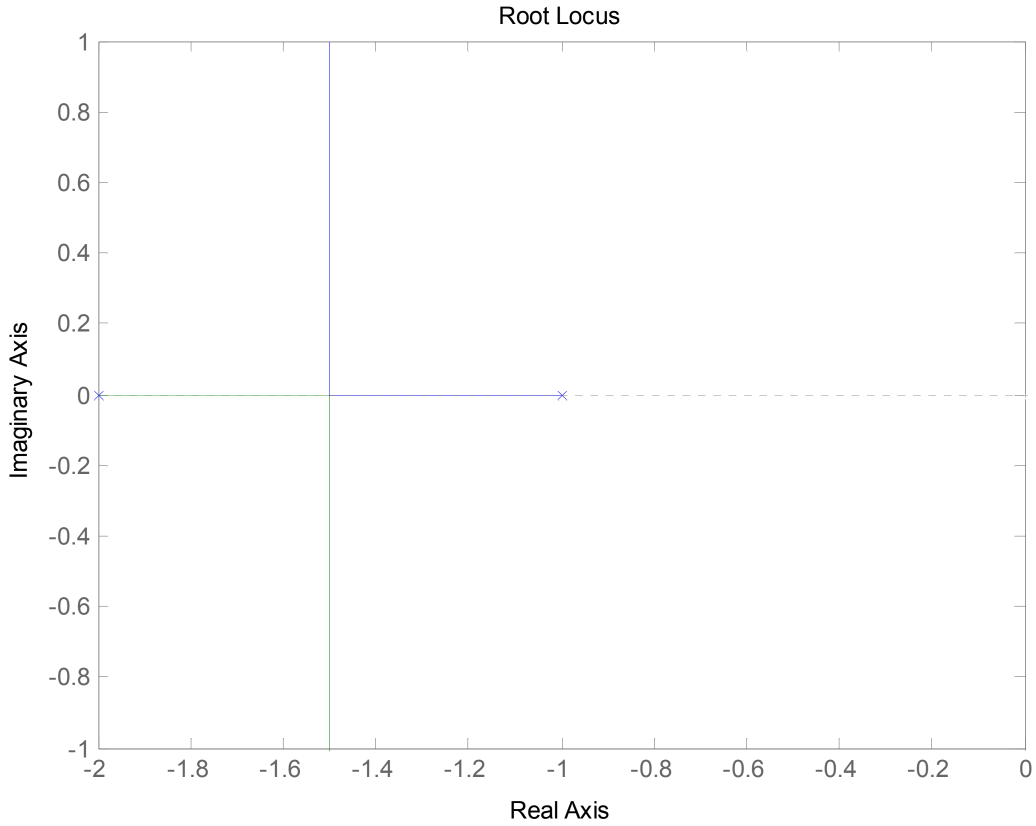



always stable

root-locus diagram

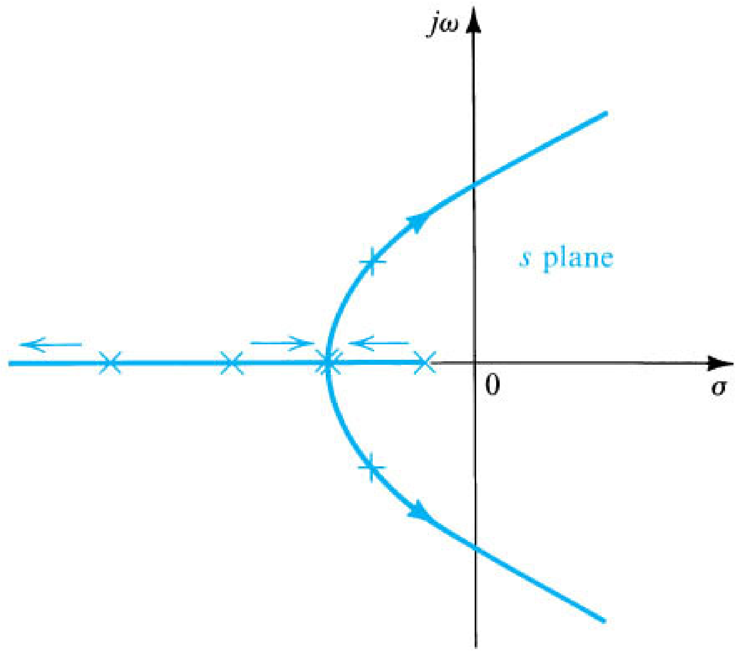
AB를 키우면서 root를 그림

```
> A = zpk([], [-1, -2], 1);
> rlocus(A)
```



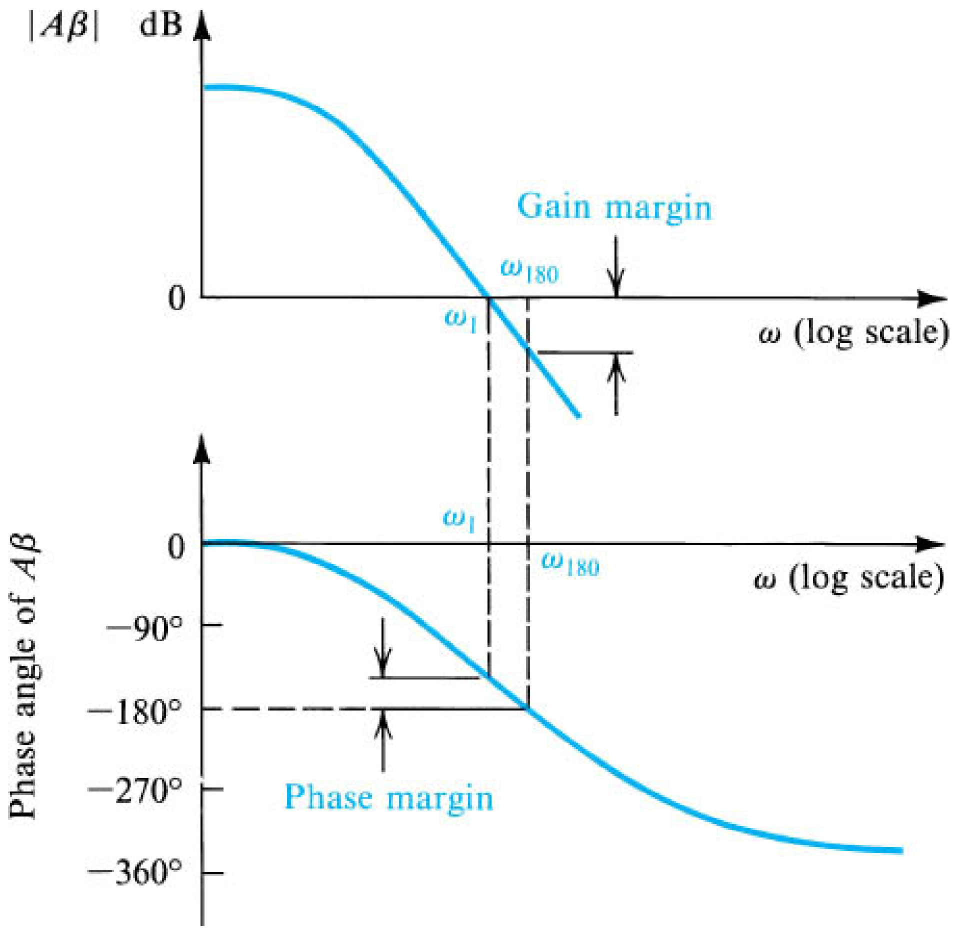
# Case 3) A(s) has more than two poles

- Unstable 할 수 있음.

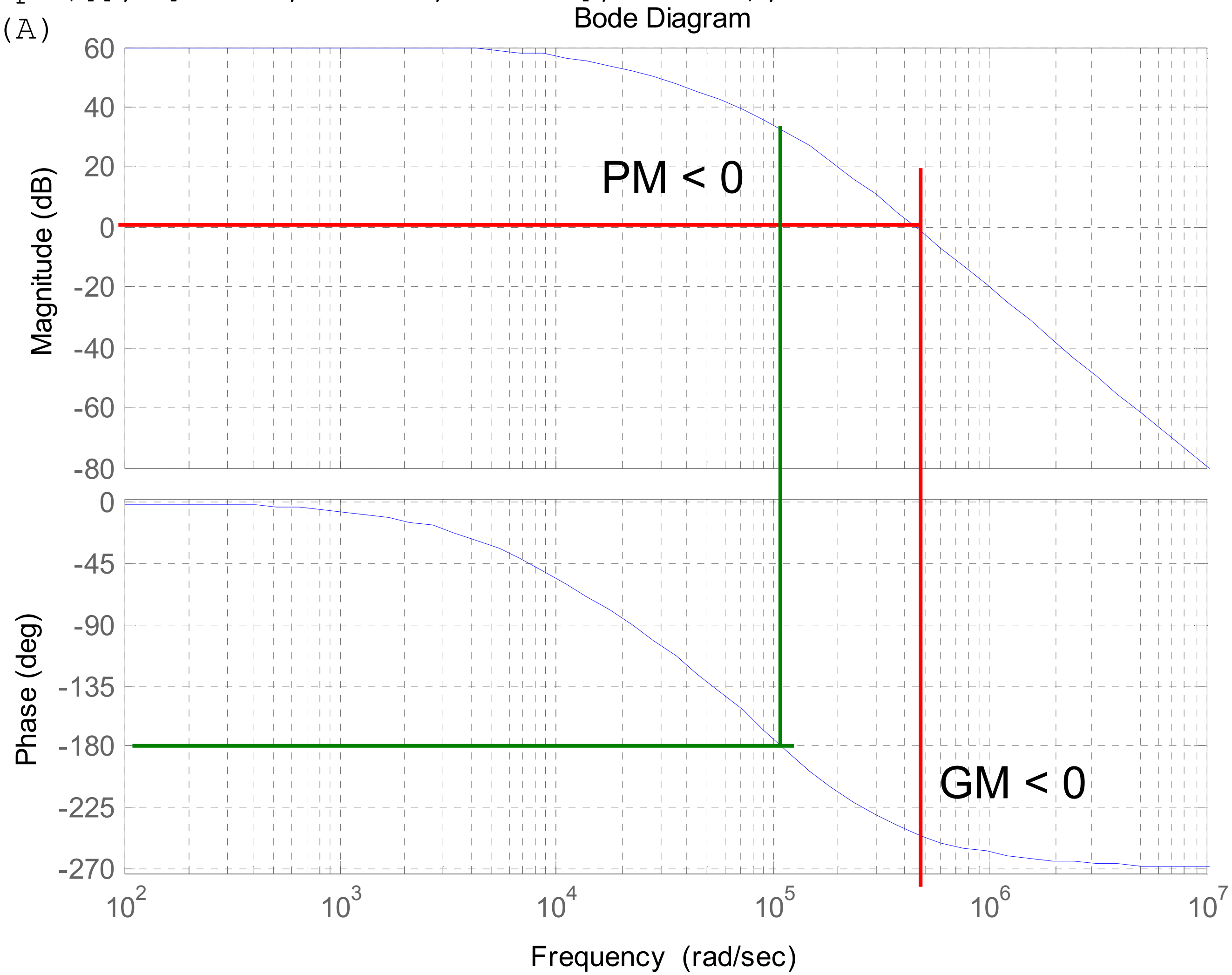


# Gain & Phase margin (Bode plot을 이용)

- Gain margin: phase가 180° 일 때의 gain의 여유
  - dB로 표시
  - GM > 0 ➔ stable
  - GM < 0 ➔ unstable
- Phase margin: loop gain이 1 (0 dB)일 때 180°에서의 phase의 여유
  - degree나 radian으로 표시
  - PM > 0 ➔ stable
  - PM < 0 ➔ unstable
- unstable일 경우 frequency compensation이 필요

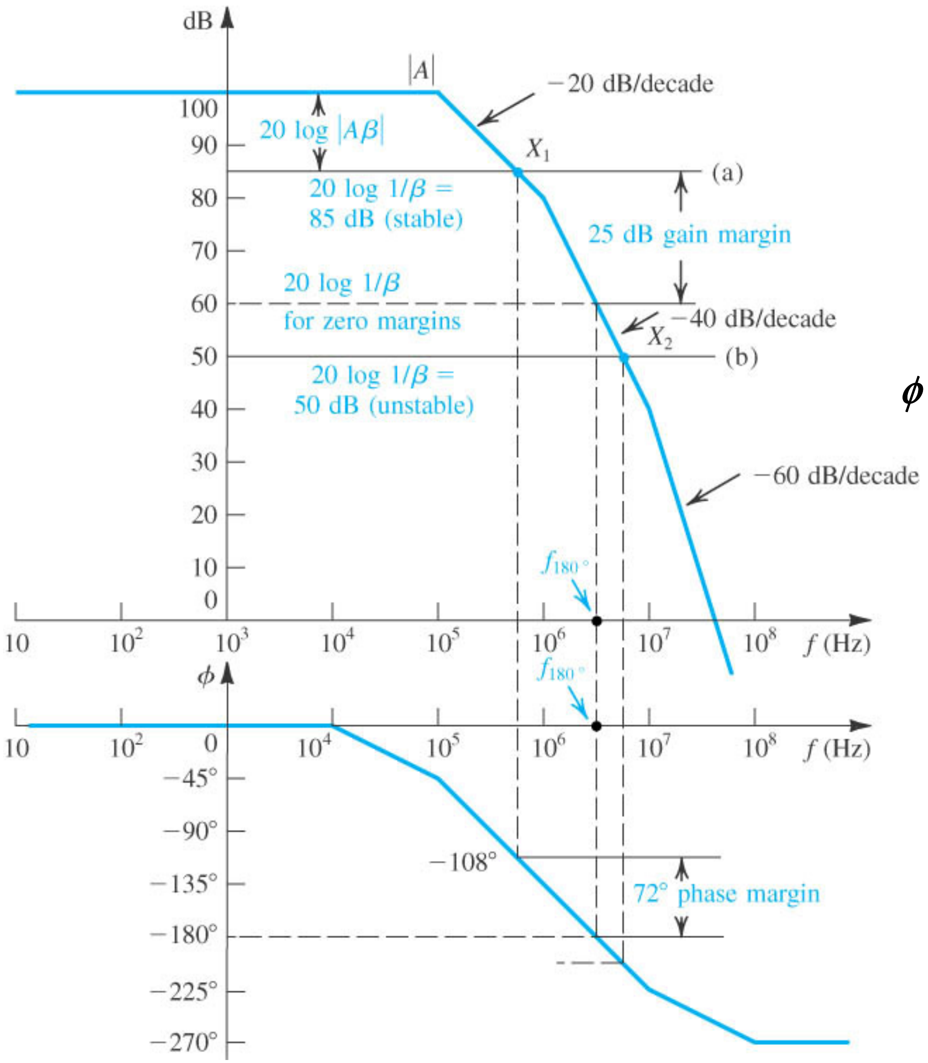


```
> A = zpk([], [-10^4, -10^5, -10^5], 10^17);
> bode(A)
```



```
>> [Gm,Pm,Wcg,Wcp] = margin(A)
Warning: The closed-loop system is unstable.
> In lti.margin at 66

Gm =
    0.0242 ➔ GmdB = -32.32dB
   ➔ B = 0.0242 = 1/41.3 ➔ GmdB = 0

Pm =
  -64.0575

Wcg =
  1.0955e+005

Wcp =
  4.5694e+005
```

# Alternative approach ($B(s) = B_0$일 때)




$$20\log\left|A(j\omega)\right| - 20\log\frac{1}{\beta} = 20\log\left|A\beta\right|$$

$$A = \frac{10^5}{\left(1+jf/10^5\right)\left(1+jf/10^6\right)\left(1+jf/10^7\right)}$$

$$\phi = -\left[\tan^{-1}\left(f/10^5\right) + \tan^{-1}\left(f/10^6\right) + \tan^{-1}\left(f/10^7\right)\right]$$

# Frequency compensation

- Negativ feedback의 장점
  - Decreased distortion, noise reduction,
  - Increased bandwidth
  - Desirable Rin, Rout
- Negativ feedback의 단점
  - Reduced gain
  - Possible instability

  ➔ 해결책: Gain을 미리 줄인다.

  (phase=180도에서 gain < 1이 되도록)

  ➔ β 를 줄이는 법

  ➔ 작은 Dominant pole을 놓는다. (pole splitting)

# Frequency compensation

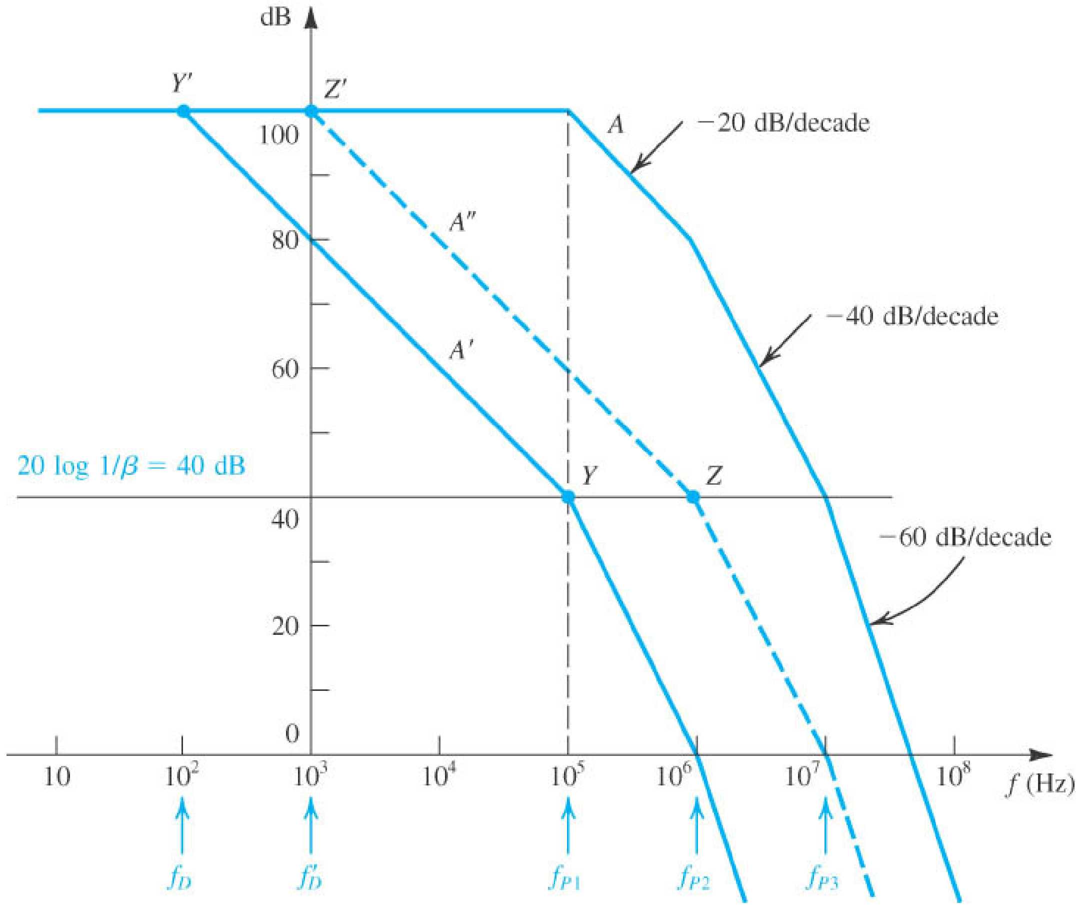
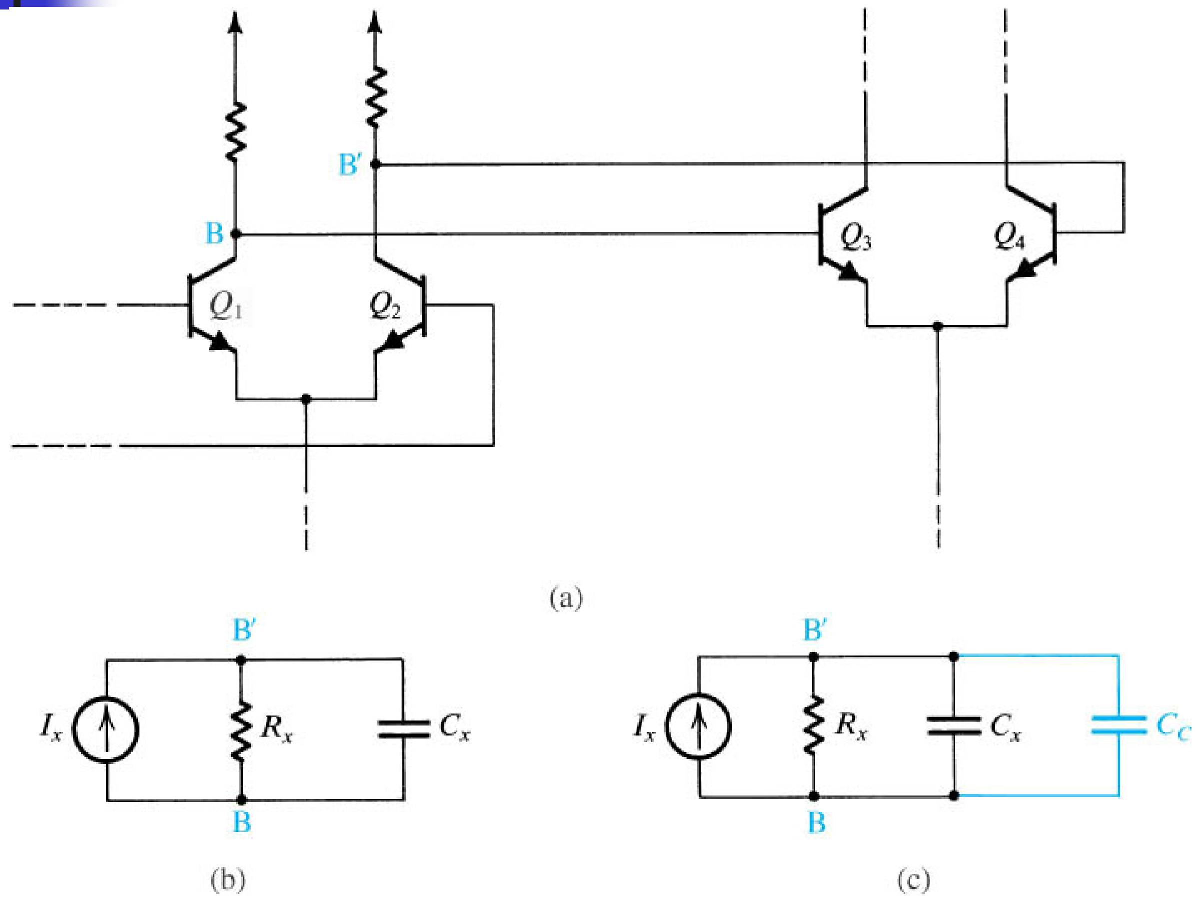



**Figure 8.38** Frequency compensation for $\beta = 10^{-2}$. The response labeled $A'$ is obtained by introducing an additional pole at $f_D$. The $A''$ response is obtained by moving the original low-frequency pole to $f'_D$.

# Capacitance in-btw the differential inputs

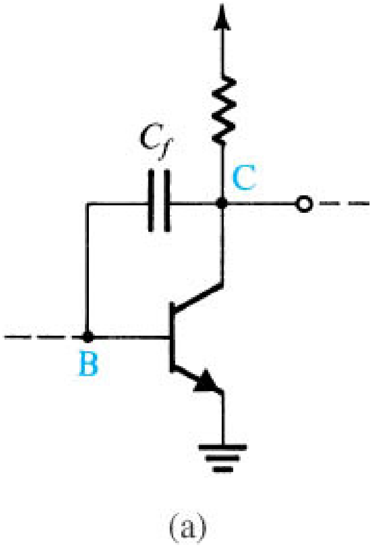



$$f_{p1} = \frac{1}{2\pi C_x R_x}$$

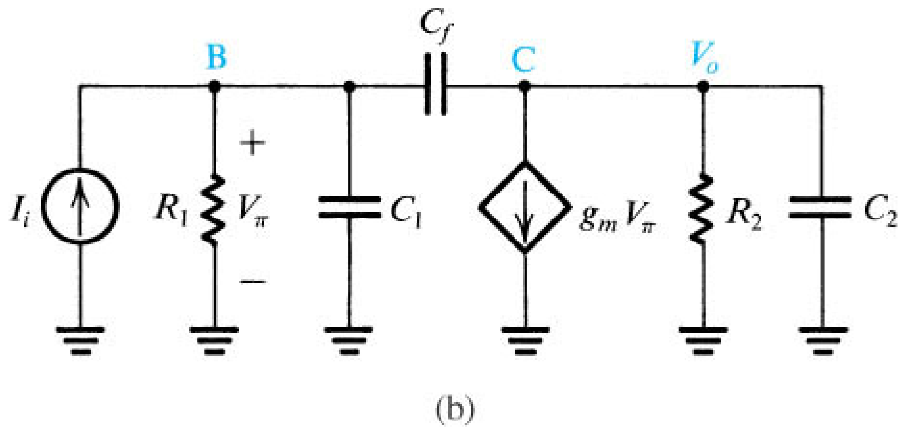
$$f_D{}' = \frac{1}{2\pi (C_x + C_C) R_x}$$

Large Cc required

# Miller compensation (feedback loop)




relatively small Cf required




Without $C_f$

$$f_{P1} = \frac{1}{2\pi C_1 R_1}$$

$$f_{P2} = \frac{1}{2\pi C_2 R_2}$$

With $C_f$

$$\frac{V_o}{I_i} = \frac{\left(sC_f - g_m\right)R_1R_2}{1 + s\left[C_1R_1 + C_2R_2 + C_f\left(g_mR_1R_2 + R_1 + R_2\right)\right] + s^2\left[C_1C_1 + C_f\left(C_1 + C_2\right)\right]R_1R_2}$$

# With $C_f$

$$D(s)=\left(1+\frac{s}{\omega'_{P1}}\right)\left(1+\frac{s}{\omega'_{P2}}\right)=1+s\left(1+\frac{1}{\omega'_{P1}}+\frac{1}{\omega'_{P2}}\right)+\frac{s^2}{\omega'_{P1}\omega'_{P2}}$$

$$D(s)\approx 1+\frac{s}{\omega'_{P1}}+\frac{s^2}{\omega'_{P1}\omega'_{P2}}$$

$$\omega'_{P1}=\frac{1}{C_1R_1+C_2R_2+C_f\left(g_mR_1R_2+R_1+R_2\right)}$$

$$\omega'_{P1}\approx\frac{1}{g_mR_2C_fR_1}$$

$$\omega'_{P1}\approx\frac{g_mC_f}{C_1C_2+C_f\left(C_1+C_2\right)}$$

# EXAMPLE 8.6

$$f_{P1} = 0.1MHz = \frac{1}{2\pi C_1 R_1} \qquad R_1 = \frac{10^5}{2\pi}\Omega$$

$$f_{P2} = 1MHz = \frac{1}{2\pi C_2 R_2} \qquad R_2 = \frac{10^5}{\pi}\Omega$$

$$f_D^{'} = \frac{1}{2\pi\left(C_1 + C_C\right)R_1}$$

$$f_D^{'} = 10Hz = \frac{1}{2\pi\left(C_1 + C_C\right)R_1}$$

# EXAMPLE 8.6(cont'd)

$$f'_{P1} \approx \frac{1}{2\pi g_m R_2 C_f R_1} \qquad f'_{P2} \approx \frac{g_m C_f}{2\pi\left[C_1 C_2 + C_f\left(C_1 + C_2\right)\right]} \qquad (8.89)$$

$$f'_{P2} \approx \frac{g_m}{2\pi\left(C_1 + C_2\right)} = 60.6\,\text{MHz}$$

$$f'_{P1} = \frac{f_{P3}}{A_0} = \frac{10^7\ \text{Hz}}{10^5} = 100\,\text{Hz}$$

$$f'_{P1} = 100\,\text{Hz} = \frac{1}{2\pi g_m R_2 C_f R_1}$$